A Model of Coupled-Maps for Economic Dynamics

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Abstract

An array system of coupled maps is proposed as a model for economy evolution. The local dynamics of each map or agent is controlled by two parameters. One of them represents the growth capacity of the agent and the other one is a control term representing the local environmental pressure which avoids an exponential growth. The asymptotic state of the system evolution displays a complex behavior. The distribution of the maps values in this final regime is of power law type. In the model, inequality emerges as a result of the dynamical processes taking place in the microscopic scales.

Key words: coupled-maps models, non-equilibrium systems, power law scaling

Inequality in the richness distribution is a fact in each economic activity. The origin of such behavior seems to be caused by the interaction of the macro with the microeconomy. Here we propose a simple spatio-temporal model for economy evolution where inequality emerges as a result of the dynamical processes taking place on the microscopic scale. That is, the microeconomy fully determines the macroeconomic characteristics of the system.

The model is composed by N interacting agents representing a company, country or other economic entity. Each agent, identified by an index $i = 1 \cdots N$, is characterized by a real, scalar degree of freedom, $x_i \in [0, \infty]$ denoting the *strength, wealth or richness* of the agent. The system evolves in time t synchronously. Each agent updates its x_i^t value according to its present state and

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the value of its nearest-neighbors. Thus, the value of x_i^{t+1} is given by the product of two terms; the *natural growth* of the agent $r_i x_i^t$ with positive local ratio r_i, and a *control term* that limits this growth with respect to the local field $\Psi_i^t = \frac{1}{2}(x_{i-1}^t + x_{i+1}^t)$ through a negative exponential with parameter a_i:

$$x_i^{t+1} = r_i\, x_i^t\, \exp(-\mid x_i^t - a_i \Psi_i^t \mid). \tag{1}$$

The parameter r_i represents the *capacity* of the agent to become richer and the parameter a_i describes the local *selection pressure* [1]. This means that the largest possibility of growth for the agent is obtained when $x_i \simeq a_i \Psi_i^t$, i.e., when the agent has reached some kind of adaptation to the local environment. In this note, for the sake of simplicity, we concentrate our interest in a homogeneous system with a constant capacity r and a constant selection pressure a for the whole array of sites.

If all the agents start with the same wealth, the index i can be removed, $x_i^t = x^t$ and $\Psi_i^t = x^t$, and the global evolution reduces to the following map,

$$x^{t+1} = r x^t \exp(-\mid (1-a) x^t \mid). \tag{2}$$

The above map can be easily analyzed by standard techniques. For $r < 1$ the system relaxes to zero and for $r > 1$ the dynamics can be self-sustained deriving toward different types of attractors. It displays all kind of bifurcations known for this type of maps [2], except, evidently, for the singular case $a = 1$. For instance, when $r > 1$ the fixed point is $x_0 = \log r / \mid 1 - a \mid$. This point becomes unstable by a flip bifurcation for $r = e^2$. For increasing r the whole period doubling cascade and other complex dynamical behavior are obtained. However, it can be shown that such evolving uniform states are unstable. When a perturbation is introduced in the initial uniform state or, in general, when the initial condition is a completely random one, the asymptotic dynamical state of the system is found to be more complex.

In Fig. 1 the after-transient spatial mean value of the field x_i is shown as a function of the capacity parameter r using a selection pressure $a = 0.8$. It can be seen that for values of $r \simeq 1$ the mean reaches a uniform, constant limiting value, but for greater values of r the dynamics becomes spatio-temporally complex.

In Fig. 2, the frequency of agent's richness in the asymptotic state of the system, is log-log plotted. As it can be seen , it scales as a Pareto like power law [3,4]. This scaling law, expressed as $P(s) \sim s^{\beta}$ with $\beta = -2.21$, is of the same type to those ones directly obtained from actual economy data [5].

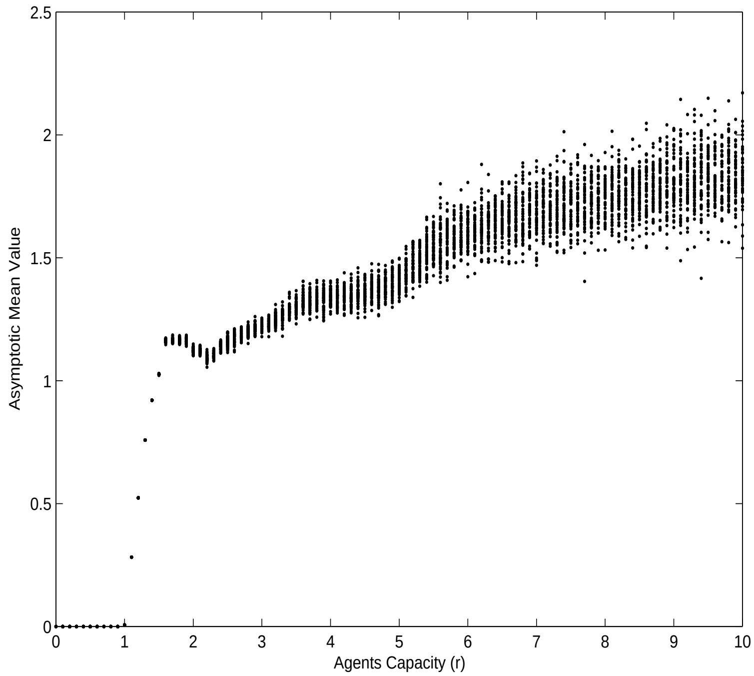

Fig. 1. Asymptotic spatial mean value of the field x_i as a function of the capacity parameter r, using a selection pressure $a = 0.8$.

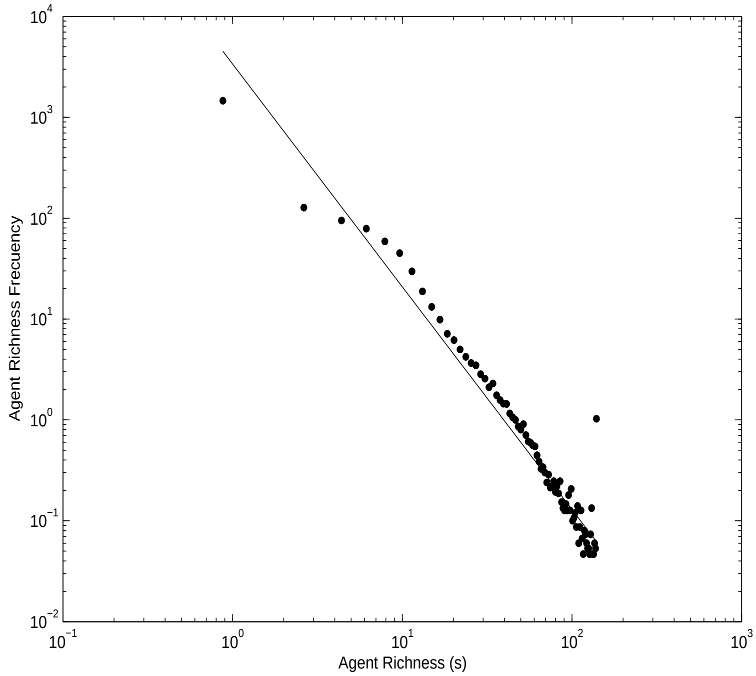

Fig. 2. Log-log plot of the frequency of agent's richness in the asymptotic state of the system.

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